An Essay

THEORETICAL AND PRACTICAL,

with

Copious and Easy Examples

on the Application of the principles of

HARMONY, THOROUGH BASS,

and

Modulation:

to the

Violoncello,

By JOHN GUNN,

Author of the Theory and practice of fingering the Violoncello.

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THE SUBJECT OF THE PRESENT ESSAY has been ever a desideratum in the study of this noble instrument. We have been, it is true, occasionally astonished and delighted with the effects which a very few of the first masters have derived from this source. Men of genius, possessing the entire command of the instrument, have been able to penetrate into the pathless region, without having, however, left a single trace of their footsteps, whereby to direct their admiring, but discouraged, followers. A humble attempt, therefore, to explore, this new, and hitherto obstructed way, and to conduct the more timid adventurer through its mazes, will, I hope, be received with much indulgence.

Some Author has observed, that no book can teach the proper use of it; and if this observation was meant to distinguish that sagacity, from the want of it, in the reader, which would direct him to the right use, or mislead him to the abuse of the book, the justness of it, while it is much to be lamented, cannot be denied. I shall however endeavour to direct the reader’s attention to the former, by submitting the following remarks, on the method I have pursued in this essay.

It must not be forgotten, that besides the science or knowledge which is to direct the bow and fingers; the production, in itself, is a practical and mechanical piece of art, which is to be acquired by pursuing, according to circumstances, either of the two following methods. The first, or analytical, is that which commences with the practical performance, and by gradual analysis, arrives at length, at the proper source, or knowledge, of principles: The second method, called the synthetical, begins with the simplest elements or principles, and proceeds gradually to their combination; the latter is that which is adopted in this essay; the former is that which was pursued by myself in the discovery, such as it is, which is now offered, with diffidence, to the world; and by which I have conducted, and would commence, pupils in their theoretical and practical study; both methods, but particularly the second, may be pursued by those conducting their own studies.

The practical or analytical method begins with the Cadence Ex: 36, No. 1, with as many of the subsequent varieties of bowing as are suitable to the learner; let the intervals taken be summed up, by $66, reduced, by $7, and inverted by $13 and 14; and the analysis Ex: 19 be clearly understood. Afterwards the construction of the Cadences Chap: 2, and their inversions, or medial Cadences Ex: 37, will introduce a more precise knowledge of the nature of intervals and their inversions, as given in the first part or theory.

The three different positions of the Chord Ex: 21, and three following examples, may then be taken, the intervals computed and inverted, as before, and compared with the theory of thorough bass, Chap: 5 and Ex: 12; and the practice and theory being further extended to the following dissonant chord of the seventh, both methods will then meet in a point of union, at Chap: 3, on the preparation and resolution of discords; where the practice proceeds, jointly with the theory, to the end of the Essay.

*We have had two instances of pre-eminence in this respect in our own Countrymen, acknowledged by all Europe. The frequenters of the Italian Opera some years ago, have often admired the unrivalled powers of one in recitative; and the praises of the other in Capriccio, are thus recorded by an English Author more than once alluded to in this Essay, who has heard a greater variety of musical excellence, with more taste and exquisite sensibility, than most other men. “It is my good fortune” says he “frequently to hear the contemporary flights of an astonishing performer on the Violoncello, which, if they could be written down and published, would not only prove a valuable treasure to the Amateurs of that manly instrument in England, but to the most brilliant professors on the continent.”*
In the course of practice, the principle, that similar intervals, are similar distances and have the same fingering in all keys, will be clearly developed; and hence it becomes altogether unnecessary to transpose the suspensions and anticipations of the last chapter; Had not the Essay exceeded the proposed length, it was intended to add another very full chapter, in illustration of the maxim §80, that "melody is harmony in succession"; to which the harmonic intervals in the circle of keys Ex: 55, was to have been the basis; but this, with the application of harmony to recitative, &c. will find a place in my School for the Violoncello, which I hope will now soon appear.

It is not pretended, nor can it reasonably be expected, in a first attempt, that the subject is entirely exhausted. I seem'd at one time, to have hit on a fundamental sequence of sevenths; but I have thro' the intervention of other ideas, now lost the chain of its application to the instrument. On this, and any other omission, I shall receive, with gratitude, any communications, made to myself, or to the Publisher. The following is a brief statement of the Contents of this essay.

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PART I.

THE GENERAL PRINCIPLES OF HARMONY.

CHAP. I. Of Intervals.

§1. The general nature of intervals, which are the distances between given sounds, always including the two terms or extremes, cannot perhaps be better introduced to the player of the violoncello, or in more familiar terms, than by supposing the following diatonic scale of notes to extend on the fourth string of the violoncello from the nut towards the bridge.

C — D — E F — G — A — B c — d — e f — g — a — b c c — d d — e e & c. 1 — 2 — 3,4 — 5 — 6 — 7, 8 — 9 — 10, 11 — 12 — 13 — 14, 15 — 16 — 17

§2. The figures placed under each letter express in a general manner, the degree, distance, or interval they represent, with respect to the primary sound C.

§3. The short line separating each letter and figure, is intended to express the greater distance or interval called Tone, and the omission of it, the lesser distance called Semitone; and hence it may be seen that the interval in every major diatonic scale, betwixt the third and fourth, and also betwixt the seventh and eighth degrees, will be invariably a semitone, and the interval between the other consecutive degrees, a tone.

§4. The term Major Scale having been mentioned, it may suffice here to observe, that the synonymous terms, major scale, major or sharp key, or major mode, refer only to the species of interval constituting the third of the scale, which is here the note E, and stands at the distance of two tones from C. It is called a major, greater, or sharp third, from being compared with the lesser interval, A—B c, or D—E F, which consisting of only a tone and semitone, are called minor or flat thirds; and consequently a scale that has for its beginning A—B C, or D—E F would be called minor scales; or the minor mode of the key of A, or D.

§5. The occurrence or exclusion of one or both semitones mentioned §3, in any interval, is the more necessary to be attended to, and their place in the diatonic scale to be always remembered, because they do not immediately appear as such, on the face of our musical notation; which would otherwise seem by its equal gradation on lines and spaces, to denote indiscriminately equal intervals between the degrees, as in the following notation of the diatonic scale §1.
§6. It may be observed that the degrees of the scale, after the seventh, are repetitions of the first seven sounds both in names and similarity of intervals. The first line of figures expresses the interval which the notes under which the figures are placed, makes with the first or lowest sound C. The second line expresses the interval it makes with the second c, and the third line, the interval it makes with the third c.

§7. The intervals after the first octave or eighth note, are called compound intervals, as being a replication of the simple intervals added to one or two octaves. These therefore may be reduced to a lower octave or more simple denomination by subtracting 7 for one octave, or 14 for two octaves, from the higher number. Thus a seventeenth may be reduced to a tenth or third; an eleventh to a fourth; a thirteenth to a sixth, &c.

§8. Thus in order to simplify the study of intervals, and of music in general, their nature and properties have been endeavoured to be reduced within the compass of eight diatonic sounds or interval of an octave, which may be considered both in an ascending progression from C to c, or descending from c to C.

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
C & D & E & F & G & A & B & c \\
8 & 7 & 6,5 & 4 & 3 & 2,1
\end{array}
\]

§9. The complement of an interval, is the quantity or interval it yet wants to compleat the octave. In the above scheme of the octave the second and seventh, fourth and fifth, are complements of each other as expressed by the figures over and under each note, which all amount to nine; and this is the rule which will afterwards be referred to for the inversion of intervals within the octave.

§10. Modulations or excursions out of the key, and other circumstances necessarily produce a change of some degree of the diatonic scale, generally by raising the fourth or some other a semitone, or depressing the seventh or some other degree of the scale a semitone, as F to F♯ or B to B♭; such being called a chromatic or minor semitone, to distinguish them from the natural semitones of the scale which are called major or diatonic semitones.

§11. These occasional chromatic changes by accidental sharps and flats, often give rise to intervals not to be found in the original diatonic scale; and when they depress an interval that
in itself minor, it gives to such the name of a diminished interval, and when they enlarge an interval that is major, it receives the synonymous denominations of an extreme sharp or superfluous interval; tho' perhaps the naturally opposite term to diminished, namely increased or augmented, would have better express its nature. The following table exhibits the intervals within the octave, with a measurement of their extent, generally by semitones, in a second line subjoined.

### TABLE of all the practicable Simple Intervals.

| Minor 2d or | Superfluous or Augmented 2d or extreme sharp 2d | Diminished 3d | Minor 3d |
| 2 tone | 1 tone | 1 1/2 tone | 1 1/2 tone |
| 2 tone | 1 tone | 1 1/2 tone | 1 1/2 tone |

Ex: 2.

| Major 3d | Augmented 3d | Diminished 4th | Perfect 4th | Sharp 4th or Triton | Imperfect or false 5th |
| 2 tones | 2 1/2 tones | 2 tones | 2 1/2 tones | 3 tones | 3 tones |
| 2 tones | 2 1/2 tones | 2 tones | 2 1/2 tones | 3 tones | 3 tones |

| Perfect 5th | Sharp or Augmented 5th | Diminished 6th | Minor 6th | Major 6th | Superfluous or Augmented 6th |
| 3 1/2 tones | 4 tones | 3 1/2 tones | 4 1/2 tones | 4 tones | 5 tones |
| 3 1/2 tones | 4 tones | 3 1/2 tones | 4 1/2 tones | 4 tones | 5 tones |

a Perfect 5th and minor Semitone.

| Diminished 7th | Minor 7th | Major 7th | Superfluous or Augmented 7th | Diminished octave | Perfect octave |
| 4 1/2 tones | 5 tones | 5 1/2 tones | 5 major & 1 minor tones | 5 1/2 tones | 6 tones |
| 4 1/2 tones | 5 tones | 5 1/2 tones | 5 major & 1 minor tones | 5 1/2 tones | 6 tones |

Three minor thirds, Perfect 5th, and major 3d, diminished 4th.
CHAP. II. Of the Inversion of Intervals.

§ 12. If one of the terms of an interval be changed into its octave, either ascending or descending, it is said to be inverted; that is, the latter interval is said to be the inversion of the former; for instance, the third C E, by being placed in the inverted order E C, becomes a sixth; the fourth C F, will become the fifth F C; the fifth C G, will become the fourth G C; the second B C, will become the seventh C B, as the following example will clearly show.

Ex: 3.

$\begin{array}{cccccccc}
\text{3rd} & \text{6th} & \text{4th} & \text{5th} & \text{5th} & \text{4th} & \text{2nd} & \text{7th}
\end{array}$

§ 13. Thus the inversion of a given interval within the octave will be its complement to 9; or in other words, subtract the given interval from 9, and the remainder will give its inversion ($\text{§9}$)

To distinguish this inversion from that mentioned §7, it is sufficient to observe, that the terms of the interval here change their order, as C E, into E C; whereas in the other they preserve the same order as C E, or C E E, a tenth or seventeenth; which is more properly an expansion of the same interval from its more compressed and simple form, to its enlarged and compound state; and moreover, the interval, in its compound state, will be of the same species that it was in its simple form; that is, the major third C E, will be a major tenth, or major seventeenth.

§ 14. But the inversions within the octave, on the contrary, will be of opposite and contrary species, excepting the intervals called perfect, viz: the octave and fifth, which invert into the perfect intervals of the unison and fourth, and contrarywise; The inversions of major intervals become minor, and minor become major; extreme, superfluous or augmented intervals become diminished, and reciprocally; as in the following example.

Ex: 4.

$\begin{array}{cccccccc}
\text{major 3rd} & \text{minor 6th} & \text{minor 2nd major 5th} & \text{major 7th minor 2nd minor 7th major 2nd diminished 7th augmented 2nd}
\end{array}$

§ 15. It may further contribute to the attaining an exact knowledge of the nature and extent of intervals, to observe, that as the octave consists of twelve semitones, so the complement of semitones that any interval contains to that number, will be the number of semitones that the inversion of such interval will consist of. Thus a minor third consists of three, and a major third of four semitones, therefore the inversion of the former, a major sixth, will have nine; and of the latter, a minor sixth, will have eight semitones.
§ 16. Before concluding this subject, it may be observed that intervals not only form the proper basis of the study of harmony, but are even its constituent parts, as syllables are of words; the easy rules given above for their inversions in one or more octaves will facilitate the computation and knowledge of them; but it must be confessed that in transposed keys where a number of sharps and flats are used, which again require double sharps or double flats to express the just intervals, they have been found, not only perplexing to beginners, but to proficients, and even Composers themselves have been led into wrong denominations. A most successful, but singular and novel method of subduing this, with many other embarrassing difficulties in musical Science, has been lately presented to the public; I mean the musical Games of Miss Young of Edin- burgh. Among several others, equally or more important and entertaining, the second Game, consisting of two parts, is an exercise on intervals, by which, a more extensive and readier use of them, however intricate their expression may be rendered, by single and double flats or sharps, may be obtained in a very few days, by the practice of a Game, the most entertaining and captivating that can be imagined, even to interesting the passions; than could be done by any other method whatsoever with the closest application, in so many months.

CHAP: III. The Natural phenomena in sounding bodies, whereon the principles of Harmony are founded, more particularly applied to the Strings of a Violoncello.

§ 17. That sonorous bodies, particularly metal and other strings, together with the general or fundamental sound, corresponding to their whole mass or length, contain and produce other sounds, simultaneously, tho' more obscurely, sounding with it, is a fact that has long been established.

§ 18. Dr. Brook Taylor in 1715, and many after him, have ascertained the most predominant of these sounds to be 1, the octave, 2, the twelfth, 3, the double octave, and 4, the major seventeenth to the original sound. What the other sounds are, that are less distinguishable among the vibrations, will be afterwards mentioned.

§ 19. The trumpet and french horn produce their lower sounds in the same succession, and even the lower sounds of the German flute, produce by increased blowing, in regular succession, on the same stop or note, its octave, twelfth, double octave, and seventeenth major, on which I have founded the small portion of harmony and modulation that is to be seen in my "Art of playing the German flute on new principles."
§20. The position of Rousseau, that, “harmony is a useless production of art”, is now more than ever considered as compleatly overthrown; and the contrary principle, that harmony is founded in nature, and that every single musical sound is a real chord, and contains its proper harmonies, is established beyond the possibility of doubt.

§21. Not to insist on the phenomenon of our English Eolian harp; an instrument has been invented in 1799 in Germany by Baron Dalberg, in which every single metal string, by the application of small rods or slips of glass, produces the perfect concord of nature (§18) in great beauty and perfection; and the whole science of practical concords and discords has been, in the following year, illustrated by Mr. Knecht, a musician of great reputation in Germany, on this principle, to which the celebrated Abbé Vogler has given the name of the pre-established harmony.

§22. Again, the converse of this phenomenon, namely, that two concordant acute sounds, generate a third, which shall be their fundamental bass, first discovered by Tartini, has been further illustrated by the Abbé Vogler, and the demonstration carried so far as to render the expences of the immense organ pipes of 32 feet in length, in future, totally unnecessary; the double bass sounds of which he has proved to be more effectually and powerfully produced by the generating principle of the upper harmonies. This and other improvements in the Organ the Abbé is about to publish in Copenhagen, under the title of his system of simplification; and other new facts relating to musical sounds under the title of Data for Acoustics.

§23. That the player on the Violoncello may know the proportions on which these natural properties of sounding bodies depend, let him consider the fourth string of the Violoncello, on which the diatonic scale $1$ was imagined to extend, to be divided into the following aliquot parts.

Ex: 5. Aliquot parts of the String.

\[
\begin{align*}
\text{C} & \quad \text{D} & \quad \text{E} & \quad \text{F} & \quad \text{G} & \quad \text{A} & \quad \text{B} & \quad \text{C} \\
\text{12th} & \quad 15th & \quad 17th & \quad 19th & \quad 21st & \quad 22nd & \quad \text{&c.}
\end{align*}
\]

Diatonic Scale -- concordant harmonics -- Diatonic harmonics

Ex: 6. Concordant harmonics Diatonic

\[
\begin{align*}
\text{Ex: 6.} & \\
\text{Diatonic:} & \\
\text{tonic:} & \text{maj:} & \text{min:} & \text{maj:} & \text{min:} & \text{maj:}
\end{align*}
\]

Since the above paragraph was written, these data, with other musical publications of a subsequent date to those mentioned in §23, have been sent me from Germany. The data contain among many other facts and observations, the Abbé's interesting defence to the effects of the music at Westminster Abbey at Handel's command, which with a variety of other information will give greater additional importance to my intended publication on the mode of conducting musical Studies with a view of arriving at greater excellence in much shorter time, than is generally thought attainable, in familiar letters addressed to the amateurs of this country. The Abbé has returned many Orders on his plan of simplification; that at St. Alksy's in Berlin, he has rendered more powerful, and great in quantity. He has added no less than one thousand six hundred of the lower number of sines.
§24. The line, Ex: 5. is supposed to represent the fourth string of a violoncello divided into the following parts; namely, the half of its length at the termination of the diatonic series C D E F G A B C, which last sound may be either taken by pressing the string firmly against the finger board, or by touching it lightly with the finger without pressing it down; in which last case, the string being set in vibration by the bow, will then give the peculiar species of sound called harmonic, as at the first minim of Ex: 6. Then let the finger or thumb be applied in the same manner, be advanced to the distance marked, which is the distance from the bridge, or $\frac{2}{3}$ from the nut, it will give the next harmonic, or twelfth to the fundamental; Advanced to the distance $\frac{1}{4}$ from the bridge or $\frac{3}{4}$ from the nut, the harmonic double octave or fifteenth will be found; At $\frac{1}{5}$ the major seventeenth, at $\frac{1}{6}$ the nineteenth, at $\frac{1}{7}$ the twenty first or flat seventh, and advancing the finger gradually, the string will produce less or more distinctly, part of the remaining diatonic intervals, marked in both examples. At similar distances from the nut or bridge, the other strings will give more or less distinctly, according to their diminished size, the diatonic and harmonic intervals of their respective fundamentals, as in the three following examples.

Ex: 7. Intervals on the third string.

Diatonic Scale of G. Concordant harmonics. Diatonic

Ex: 8. Intervals on the second string.

Diatonic Scale of D.

Ex: 9. Intervals on the first string.

Diatonic Scale of A.

§25. The aliquot parts from the nut to the half of each string, coinciding with the diatonic intervals of each scale are noticed, not with any view to practice being guided by any such
geometrical measurement of the intervals; as the cultivation of the ear, and its means of judging of the accuracy of sounds, depend on perceptions and principles altogether different; but to show generally that the same laws and proportions take place in the lower and acute octaves. To the latter I have ventured to give the name of diatonic harmonics, merely from their peculiarity of sound, and to point out a difference in two of the intervals marked with an asterism, from those received in practice, which distinguish the same intervals of the trumpet &c. namely, that the fourth of every scale is a little sharper, and the sixth somewhat flatter than our practice.

§ 26. The sound produced by \( \frac{1}{7} \) of the string, which has been recently called with much propriety the boundary betwixt concordant and dissonant harmony, has been commonly thought too flat, which may be conceived from the observations that will next follow, to be in some circumstances true, without its being inconsistent with its justness in forming the interval of a minor seventh, in the above proportion, wherein it generates its proper fundamental (§ 22).

§ 27. Intervals have been hitherto supposed to consist of only two species of elementary sounds, a tone, and semitone, but the production of pure harmony renders a further distinction of the former interval necessary as at Ex. 6. namely a tone major, of which there are three within the octave; viz: between the key and second, the fourth and fifth, and between the sixth and seventh of the scale, of which the ratio or proportion is 8 to 9. and secondly a minor tone, of which there are two within the octave, the first between the second and third, and the second between the fifth and sixth of the scale, expressed by the ratio of 9 to 10, which is the small interval called commaless than the former, a difference expressed by the proportion 80 to 81, which is judged to be about the eighth part of a tone. Thus the more accurate division of an octave is into three major tones, two minor tones, and two semitones.

§ 28. Rousseau however has asserted, and after him other authors and compilers of Encyclopaedia, that so small an interval as comma, cannot be distinguished by ears such as ours, and can only be appreciated by calculation. In order to examine the truth of this assertion, and to judge of the extent or existence of the small interval in question; let a violoncello be perfectly well tuned, and the note E, of the following example at k, be stopped so as to produce a major sixth to the open string G; then let the same E be sounded with the open string A, as in the example;
if the latter sounds appear to the performer a perfect fourth, it will be impossible to prove the existence of our small interval to an ear in that state of cultivation; but should he be led by any sensation or desire to make his E somewhat sharper, as signified by the direct, in the example, to produce a pure chord, he may rest satisfied that his feelings are conformable to the most rigid theory; for the distance he has moved from the first station of the finger is comma; the second station being a tone major from the open string D, and the former a tone minor.

Ex. 10. Intervals measured by Tones major, Tones minor, and Semitones.

<table>
<thead>
<tr>
<th>Maj.</th>
<th>min.</th>
<th>sem.</th>
<th>M. m. s.</th>
<th>M. m. s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-2</td>
<td>2-2</td>
<td>3-3</td>
<td>2-1-1</td>
<td>6-3-3</td>
</tr>
<tr>
<td>h</td>
<td>i</td>
<td>k</td>
<td>m</td>
<td>n</td>
</tr>
<tr>
<td>2-1</td>
<td>1-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

§29. For at h is shown that the interval of a ninth between c and d, is four major tones, two minor tones, and two semitones, namely one major tone, the true interval of the second of the scale which the ninth represents, added to the octave (§27) whereas at i, the major sixth, two major tones, two minor tones, and one semitone, being added to the perfect fourth, one major tone, one minor tone, and one semitone, make only three major tones, three minor tones, and two semitones; which is less than the interval at h, by the difference between a tone minor and a tone major, which is comma. Therefore as the open string cannot descend a comma, the first finger must advance that small interval to it: the transposition of the same two intervals at k, is only for the sake of perceiving a more distinct vibration from smaller strings; and these again repeated an octave higher at l, and m, is for the purpose of making the experiment with the thumb on A, the octave of the open string, whereby it may be made to descend a comma, if the first finger on E, keeps its station; or the experiment may be reversed as at m, beginning with making the perfect fourth, which will occasion the thumb on G, to advance a comma to make a major sixth with E, and consequently it will no longer be a perfect octave to the open string G.

§30. Again, the intervals of three fifths at n, added together as a thirteenth from C to n, at o, amount to six major tones, three minor tones, and three semitones; but this interval, taken as a perfect octave, and major sixth to C, as at p, should consist only of five major tones, four minor tones, and three semitones, which is a comma less than the interval measures at o; consequently...
ly if the note A, be taken with the second finger, exact unison to the open string, and then
made a major sixth to C, taken with the first finger, as at p. such C, will not be a just
octave to the open string C; or conversely, make C with the first finger, a just octave to
the fourth string; then make a major sixth to it with the second finger on a, this last will
be a comma flatter than the a of the open string.

§ 31. It would however be rather fastidious in any one, hence to imagine, as it would
be unjust to assert, that good performers could be proved frequently out of tune on the violin
or violoncello, merely because in a rapid succession of notes, their finger might be stopped
suddenly and demonstrated to be a comma too sharp or flat. In such a case, to speak
with Rousseau, the difference might be said not to be appreciable by the ear; but in sounds
that are sufficiently long for the ear to perceive their agreement or disagreement, the error
is rectified before it can well be perceived; for on my hypothesis (§29) of the passage at
1 and m, one or other of the terms will descend or ascend to the just interval, by making
up its complement: and on this principle, namely, that a just ear will always aim at ac-
complishing the perfection of an interval, it is, that if a less accurate performer in taking
an interval, which I shall suppose represented by the number 20, of which his part ought
to be 12, should only make it 11\(\frac{1}{2}\), the more cultivated ear, who has to play the other part,
which should naturally be 8, will make it 8\(\frac{3}{4}\). And thus it seems to be out of the power of
any one to be for any length of time in tune, in playing concordant intervals with another
part that is frequently out of tune; not unlike the attempting to play just intervals on strings
on which true fifths cannot be taken.

§32. What has been said above concerning diatonic intervals at both ends of a
string, may be considered as digressive from the chief point of enquiry, which was to
consider the concordant harmonic intervals, and the aliquot parts of strings producing them;
and from the premises established, the following inferences may be drawn.

§33. First, that the vibrations of strings being in the inverse ratio or proportion
of their lengths, the vibrations of the concordant harmonics belonging to and arising from
the fundamental sound, are in the following proportion to each other, beginning with
the fundamental, and proceeding with its octave, twelfth, double octave, seventeenth
and nineteenth, viz. as 1, 2, 3, 4, 5, 6, which are respectively in their order a series
of the nearest and simplest possible relations of vibration.

§34. Secondly. That any two of these intervals after the first, sounding togeth-
er, will generate a third sound, which is their fundamental, and always corresponds
to the difference between their ratios. That is, 1, 2, 4, being inversely the lengths of string,
and the direct ratios of the vibrations of the fundamental sound, its octave and double octave;
2, 3, is that of the fifth, or interval formed by the octave and twelfth; 3, 4, is that of the
fourth, or interval formed by the twelfth and double octave; 4, 5, is that of a major third,
or interval formed by the double octave and seventeenth; 5, 6 is that of a minor third, or
interval formed by the seventeenth, and nineteenth: all which intervals refer to one and
the same generating fundamental sound, as will appear by taking the difference of their
ratios, or subtracting the lower from the higher term.

§35. Thirdly, the ratio 6, 7 being less than the minor third, forms together
with the fundamental and other concordant intervals a less perfect species of harmony,
hinted above §26, and will be mentioned on another occasion.

§36. Fourthly, the series 4, 5, 6, or fifteenth, seventeenth and nineteenth are
the degrees of the greatest compression or approach that concordant intervals will admit
of, and these being inverted or reduced to their lower octave §7, they become the first
or fundamental, major third, and the perfect fifth, of the diatonic scale.

§37. Lastly, the series 1, 2, 3, 5, omitting 4 as a second replication of the
fundamental; or the fundamental, its octave, twelfth, and major seventeenth, affords
the greatest variety in compass and concordant intervals of which four sounds are capable.
§ 38. The position of the harmonics of $C$ § 37, may be taken on the violoncello,
by placing the first finger on the third string, octave to the fundamental or open string
$C$, which finger will also take the twelfth on the second string, and the third finger on
the first string will give the major seventeenth, as at $h$, in the following example.

Ex: 11. The perfect Concord of natural phenomena,
reduced into various positions.

§ 39. The intervals $17^{\text{th}}, 12^{\text{th}}$, and $8^{\text{ve}}$ are compressed at $i$, by the octave being
placed as fifteenth or double octave, and the intervals descend at $k$, $l$, $m$, and $n$, under
different denominations or distances from the fundamental, till they make with it at $o$, the
third and fifth. But it may be observed that the arrangement at $i$, is similar to that at
$m$; that at $k$, to that at $n$, and that at $l$, is similar to that at $o$; making in fact only three
essentially different positions or arrangements of the notes $C$, $E$, $G$, or first, or octave
third, and fifth, in different octaves, which for the sake of simplification, are all consi-
dered in thorough bass, as making these three intervals with the bass, in whatever octave
or position they may be placed as at $h$, $i$, $k$, in the following example, where either of
the three positions taken with the bass, is called indiscriminately, the perfect chord
of $C$. It is called the chord of $C$ major, from its third $E$, being a major third to the
fundamental; it is also called the common chord, and by some few the Triad. But when
either of the other parts $E$, or $G$, are taken as a bass, then the whole chord is said to be
inverted, and the former intervals assume different denominations, forming two distinct chords, as at 1 and m, although they are in fact only inversions of the original chord, having the same C, for their fundamental bass, tho' it does not appear in the lowest part.

Ex. 12. The perfect major.

\[
\text{chord of C.} \quad \text{The chord of the } 6^{th} \quad \text{The chord of the } 4^{th}
\]

Harmonics, which may be one or more octaves higher.

§40. With respect to the Perfect chord, it may be observed, First, Its signature may be 3, 5, 8, as above, or any one or two of these figures; but for the most part it is known and distinguished from other chords, by its having no figures: it has on many occasions a sharp (♯) or natural (♮) above or under it, which never applies to any other interval than its third, and in all cases of change or doubt, serves to denote whether such third be major, which commonly must be inferred from the clef or key, and Secondly. The intervals of which it consists are as follow: the third from the bass, being its characteristic (§39) must be major; the interval formed by its third and fifth, is a minor third (§41) its fifth is perfect, and the interval between its fifth and octave, which (§41) is a fourth or complement to it, and must be a perfect fourth (§14) as being the inversion of a perfect fifth.

§41. With respect to the chord of the Sixth, it may be observed, First, That its most usual signature is 6, tho' it may sometimes have 3 or 8 added, expressive of its other intervals. Secondly. That its bass is the third of the original or fundamental chord, which being major makes its inversion the sixth to be minor (§14) and its third is minor being the interval from the third to the fifth of the original chord (§40 and 34.)
§ 42. With respect to the chord of the fourth and sixth, it may be remarked,
First. That its signature is always that of these two figures. Secondly, that its bass is
the fifth of the original chord; That its fourth is perfect, being the inversion of the per-
fect fifth, and that its sixth is major, as being the inversion of the minor third, between
the third and fifth of the original chord.

§ 43. Not to lengthen out this Essay by inconclusive conjecture, or hypothesis,
on the origin of a minor perfect chord, I shall hasten to the more useful and practical part,
by remarking that it is in every respect similar to the major chord, above analysed, except-
ing as to its characteristic, the third, which is minor; and consequently, that no changes can
take place in any other interval; or in those of its two derived chords, the sixth, and fourth
and sixth, excepting such as arise from that interval and its inversions.


§ 44. The signatures being the same as in the major species, its characteristic, the
minor third, is known by the clef, as above, or by an occasional flat or natural put over or
under the bass. The interval betwixt its third and fifth, is a major third, and consequently
the inversion of that interval in the chord of the fourth and sixth, is a minor sixth. The sixth of the
second chord, being the inversion of the original minor third, is major; and in a word all thirds and
sixths are here the converse of those intervals in the major chord and its two inversions.

Of the Chord of the minor or flat seventh,
and its three inversions or derived chords.

§ 45. Of the uses, with a more detailed analysis of the intervals, of this chord.
particular mention will be afterwards made in the practical part; it may suffice at present to say, it consists of an additional interval of a minor third (§26) added to the major third and fifth of the perfect major chord. That it may be conceived as having the fifth of a major or minor key (§4) either actually existing or about to take place, for its fundamental, to which the fourth of the key by inversion (perhaps with a little temperament §29) will be the interval of a flat seventh. It has four positions of its upper parts or harmonics, as at h, i, k, l, and three chords derived from its inversions, as at m, n, o, of the following example.


§46. Thus in any of the four positions h i k l, B is esteem'd the major third, D, the fifth, and F the minor seventh to the fundamental G, all implied by the signature 7.

§47. The chord of the fifth and sixth, called also the chord of the false fifth, has B the third of the original chord in the bass. It consists of a minor third D, an imperfect flat, or false fifth F, and a minor sixth G, the fundamental of the original chord.

§48. The chord of the third and fourth, called also the chord of the greater sixth, has D, the fifth of the original chord for its bass, to which F, the original flat seventh, is a minor third, G the original fundamental is a perfect fourth, the inversion of the fifth, and B the original third, a major sixth.
§49. The chord of the second and fourth called also the chord of the second and chord of the Triton or sharp fourth, has F, the minor seventh of the original chord for its bass, G the original fundamental for its major second, the inversion of the flat seventh, B, the triton or sharp fourth, the inversion of the false fifth, and D a major sixth.

RECAPITULATION and CONCLUSION.

§50. From what has been stated, it may be easily inferred that the preceding chords, are reducible to two, which are called with peculiar propriety Fundamental Chords, as being those, on whose basis all other varieties take place; namely, the Concordant perfect chord, of two species, the major and the minor, and the fundamental discord, or the less perfect chord of the minor seventh. The latter has from its peculiar qualities attracted the notice of all practical musicians; Geminianni has resolved discords by it; The Abbé 

calls it the entertaining chord. The ingenious Mr. Knecht aptly stiles it, the boundary between concord and discord, and the most pleasing and favorite passages of modern melody are derived from its intervals. A species of it with a minor third is a more decided discord as at h i k l, and a third species called the diminished seventh, which, with its inversions, have been also called with much propriety, chords of substitution or borrowed harmony, as at n o p q, of the following example, answering in a great measure the purposes of the first species, to which it may be conceived for the first time to be substituted, as will appear by its employment in the practical parts.

Ex: 15. Chords of the minor 3d and 7th

and Inversions.

Chords of the diminished 7th

and Inversions.

Ex: 15. Chords of the minor 3d and 7th

and Inversions.
§ 51. There are however species, as musicians call them, tho perhaps the term variety, would be a more philosophical distinction, of both fundamental chords, arising from accidental changes of their intervals by passing notes or otherwise, which occasion together with their inversions a variety of intervals (Ex.:2.) not to be found in the natural diatonic scale.

§ 52. There is still another class of chords, admitted in theory and practice, which are considered as extending beyond the limits of the octave, called originally, chords by supposition, from placing the bass by thirds below. Of these the chords of the ninth, and eleventh, as established by Rameau have been longest known and taught in this Country; to which Marpurg, whom all the Harmonists of Europe consider as the greatest modern theorist has added the chord of the thirteenth, constituting them a distinct class, by the name of chords of the second rank. Marpurg's numerous profound and scientific writings having never been translated into English, his system has been only known here thro' the medium of the classical compositions which conform to it, and by means of two eminent German Masters* resident here, who have formed their pupils on its principles. A late writer on thorough bass, a German, of a very inferior class to his two countrymen, in endeavouring to introduce into theory, an unadmitted innovation of Kirnberger's, who in other respects is justly accounted a great master of harmony, has most unwarrantably accused two late writers of this Country, of having "revived a confused and obsolete system," merely for their having followed Rameau and Marpurg in these Chords, as will appear from the following example.

Ex: 15. Theory of the construction of the Chords of the 9th, 11th and 13th according to RAMEAU, MARPURG, KING and SHIELD.

* Messrs. Baumgarten and Dietzenhofer.
§ 53. The assertion therefore, that the system is obsolete, so far as it respects this
Country, is evidently false; and with respect to Germany, one of the greatest and most admired
masters in Europe, who by a late tour in Germany has happily re-established his health, ob-
served on this subject, that were an Englishman to ask any man of science in that Country, whether
the Germans looked on the system of Marpurg to be confused and obsolete, he would probably
make no other reply than asking another question, namely, whether the system of Newton
was looked upon as confused and obsolete in England; and such I can assert to be the prevail-
ing system from the completest series of German publications on the Science I can procure
from the continent; out of which I select the following example from the new system of Mr. Knecht
(§ 22) which Abbe Vogler in a late publication mentions as having now cleared up what was left
unascertained by Marpurg and Kirnberger.

Ex. 16. Construction of the Chords of the 9th, 11th, and 13th
according to Knecht's new system.

<table>
<thead>
<tr>
<th>Chord of the compound simple major 9th</th>
<th>Compound simple minor 11th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chorus simple major 9th inversion. 9th &amp; 7th</td>
<td>Chorus simple minor 11th inversion.</td>
</tr>
<tr>
<td>Chorus compound 11, 9, &amp; 7. simple major 11.</td>
<td>Chorus 13, &amp; 7.</td>
</tr>
</tbody>
</table>

§ 54. It may be remarked that of all the different modes of constructing the above
chords, that of Mr. King, which he has much reasoned upon in his "General treatise on music"
before he ventured to differ from the great authorities of Rameau and Marpurg, agrees most
with Mr. Knecht, in assigning the same fundamental concord and minor seventh, generally to
all the chords. Their works appeared nearly at the same time, that of Mr. King in London in
January 1800, and that of Mr. Knecht in Germany in February, the same year.

- Mr. John Cramer.

END of the FIRST PART OR THEORY.
PART II.

The APPLICATION of the PRINCIPLES of HARMONY to the practice of the VIOLONCELLO.

CHAP: I. Of the manner in which all the fundamental chords and discords with their inversions, are to be taken.

§55. Harmony, with respect to the greater, or smaller, intervals in which it may be displayed, has been divided into two species, namely the compressed, when the intervals so approximate as to leave no space for an interposing concordant interval, and the expanded or dispersed, consisting of intervals so large, that another concordant interval might be interposed between their extremes. Of the first species, which may be called the more artificial, serving the purposes of simplification of the intervals, and ease of execution on key'd instruments, the following will give an idea; being those practicable on the Violoncello.


§56. Of the intervals constituting expanded or dispersed harmony, which may be called the more natural, variegated, and rich species, §37 in compositions for different voices and instruments of extended compass, the following may serve as examples.

Ex: 18. Chords of expanded or dispersed Harmony.

§57. All the intervals in this example, extending beyond the octave, are easily reduced into their corresponding intervals in compressed harmony, by subtracting the number 7 from the former §7. Thus at o, the eleventh is reducible to a fourth, which makes the chord at o, to be a fourth and sixth to the lowest sound C; the fourteenth at s, is reducible to a seventh, which makes the chord, a fourth fifth and seventh, or chord of the eleventh to C. The short oblique lines at n, o, q, r, and s, t, point out the solution of the discordant intervals, which will afterwards be more particularly explained.
§ 58. It is evident that the compress'd chords Ex: 17, cannot be transposed or taken in any other key than those mentioned. These may therefore be considered as a class by themselves, which may be occasionally used, but will be taken no further notice of in this essay. Those again of the second species of harmony at Ex: 18, are capable of being taken in all the other keys, but on account of one or two open strings being used in each, they cannot serve as a model of fingering or general rule for taking the chord, but must for that reason be considered as a class by themselves, and an exception to the general rule now to be established.

§ 59. In order to ascertain what this general rule shall be, let the chord of D, both major and minor be analysed, and inverted, the major thirds into minor sixths, and the minor thirds into major sixths as at h, and i, in the two following examples; a chord will appear at k, consisting of such two sixths, forming the intervals of sixth and eleventh or fourth by inversion, to the lowest sound A, to which if the fundamental sound D, be added as at l, in both examples, a chord will be formed consisting of a fifth and two sixths, or fifth, tenth, and fifteenth, which again are reducible to the third, fifth, and octave to the lowest sound D, by


of the Perfect major chord. of the Perfect minor chord.

§ 60. Again, if each of the harmonics at h, in the two following examples, be taken in succession, as the lowest sound of a chord, the three positions at i, k, l, of both examples, will arise; the intervals whereof being reduced, will be third, fifth and octave, to the figured bass in the subjoined line. But if the third, and fifth of the chord, be put to the subjoined bass as at m, n, the intervals at m, will be the third and sixth, and those at n, will be the fourth and sixth to the Bass, if reduced to their compressed form. See § 39 Ex: 12.

Ex: 21. Three positions of the major chord of D. Ex: 22. Three positions of the minor chord of D.

Ex: 23. Three positions of the chord of G. Ex: 24. Three positions of the chord of C.

Ex: 25. Three positions of the chord of F.

Ex: 26. Three positions of the chord of E.

Ex: 27. Three positions of the chord of C.

Ex: 28. Three positions of the chord of F.

Ex: 29. Three positions of the chord of B.

Ex: 30. Three positions of the chord of G.

Ex: 31. Three positions of the chord of F.

Ex: 32. Three positions of the chord of E.

Ex: 33. Three positions of the chord of C.
§61. Hence any note in the bass having no signature, or any one or more of the figures 3, 5, 8 ($§40) being taken by the first finger, will have its proper accompanying harmonics on the two other strings, as in the first position of the two following examples, which in a major chord, I would call the leading position, for reasons which will afterwards appear; a bass note, with the signature 6, by placing the first finger on it, will have its proper harmonics, by placing the second finger on the other two strings, as in the second position, which is called Medial, and a bass note having the signature of the sixth and fourth, by placing the first finger on such note, will have its proper harmonics on the other two strings, as in the third position, which is called final.

Ex: 23. The major chord, the 6, and $\frac{3}{4}$. Ex: 24. The minor chord, the 6, and $\frac{3}{4}$.

<table>
<thead>
<tr>
<th>First or</th>
<th>Second or</th>
<th>Third or</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

§62. The chord of the minor seventh $\frac{3}{4}$ Ex: 14, having $C$ natural added to the above major chord, is together with its inversions taken on the same three positions, forming on the first, the chord of the seventh, which in this position I would also call the chord of the tritone or sharp fourth, from its characteristic interval. On the second position, the chord of the fifth and sixth, by an extension of the fourth finger. On the third position, the chord of the third, and fourth, which I would also call the chord of the false fifth, from its distinguishing interval. And lastly the chord of the second and fourth may be taken, by placing the fourth finger on the bass note, and adding the other fingers as in the first position, pointed out by the directs, or by placing the second finger on the octave of the bass note, adding the other fingers as in the third position.

Ex: 25. Chord of the minor seventh of D, and its three inversions.

<table>
<thead>
<tr>
<th>Intervals of the full Chord</th>
</tr>
</thead>
<tbody>
<tr>
<td>First position</td>
</tr>
<tr>
<td>or Tritone</td>
</tr>
</tbody>
</table>

§63. In the same manner all the chords in the system may be taken, beginning with the open string $C$, and proceeding along that string by semitones, placing the first finger successively, for each fundamental chord, on $D$, $B$, $F$, $E$, $F$, and $G$, which I call the first series; then taking the remaining five fundamentals, $A$, $A_{b}$, $A_{b}$, $B_{b}$, and $B_{b}$, the second series, on the third string, as in the following Tables.
Table of all the Major Perfect Chords, with their Inversions, the Chords of the Sixth, and Fourth and Sixth.

First Series. taken on the fourth, third, and second strings.

Second Series. taken on the third, second, and first strings.
Ex: 27. **Table of all the Minor Perfect Chords, with their Inversions**

the chords of the sixth, and fourth and sixth.

**First Series**, taken on the fourth third and second strings.

<table>
<thead>
<tr>
<th>Chords of C</th>
<th>No. 1</th>
<th>No. 2</th>
<th>No. 3</th>
<th>No. 4</th>
<th>No. 5</th>
<th>No. 6</th>
<th>No. 7</th>
<th>No. 8</th>
<th>No. 9</th>
<th>No. 10</th>
<th>No. 11</th>
<th>No. 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 C♯</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Eb</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 E♭</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>6 F</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>7 F♯</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 G</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Ex. 28. **TABLE of all the Chords of the fundamental minor seventh, and its three Inversions.**

**FIRST SERIES** on the fourth, third, and second strings.

<table>
<thead>
<tr>
<th>Chords of</th>
<th>No. 1</th>
<th>No. 2</th>
<th>No. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 C#</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 D#</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Eb</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Eb</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 F#</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 G</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**SECOND SERIES** on the third, second, and first strings.

<table>
<thead>
<tr>
<th>No. 9</th>
<th>No. 10</th>
<th>No. 11</th>
<th>No. 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 Ab</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Ab</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 Bb</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 Bb</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Annotations on the foregoing examples.

§ 64. In order to form a proper position of the hand for taking the chords, the knuckles of the fingers ought to be raised, and their points bent downwards, so that they may be somewhat round, and the four fingers together, resemble the form of a gothic arch, as represented in the drawing at fig. 16, page 7 and minutely described § 1 of my theory and practice of fingering the Violoncello excepting in the second position of the chords, where the second finger must be laid flat on two strings, and at right angles with them; in which direction alone the interval of a perfect fifth can be found, as represented by the dotted lines in the first part of the following Ex. 29, where the other dotted lines at h, i, k, l, point out the inaccuracy that would ensue from an oblique position of the hand, or tendency of the fingers. The lines of this example are intended to represent the four strings of a Violoncello, and the larger black dots, the place of the fingers on the strings, in the three positions of the major chord of D (Ex. 3, Ex. 26). The musical characters whereof may be seen under each position, at Ex. 30.

Ex. 29. Representation of the three positions on the strings.

§ 65. Thus the eight chords of the first series, major and minor Ex. 27 and 28 may have their third position in the same place where the first position is taken; or they may be joined together, constituting a chord of the fundamental, its fifth, tenth, and fifteenth as at h, in the last example. And in the same manner the third position of the minor seventh of the eight chords of the first series Ex. 28 may be taken at, or joined to the first position as at i, k, making the full chord of the fundamental, its fifth, tenth, and fourteenth as at l.

§ 66. The third position of the first five chords, Ex. 26, 27, 28, may be taken both on the fourth, third, and second strings, and at the first position, on the third, second, and first strings, which (together with the second position when it ought to be taken on the last mentioned string) is marked in these examples, 1, meaning that the first finger is to be put on the third string, as 1 denotes that the first finger is to be placed on the fourth string: and it is to be observed that the first position of § 8, and the following chords may be taken both ways.
§67. The first or leading positions of C, D, E₄, E₅, and F, both in Ex. 26 and Ex. 28 must be taken an octave higher than at Nos. 1, 3, 4, 5, 6, when they are followed by the final positions of F, G, A♭, A♯, and B♭, either major or minor; as in the two following examples of final Cadences in the five latter keys. What the nature and properties of these final Cadences are, will be explained in the next chapter.

**Ex: 31. FINAL CADENCES in major keys.**

![Diagram of final cadences in major keys]

**Ex: 32. FINAL CADENCES in minor keys.**

![Diagram of final cadences in minor keys]

§66. As a readiness in computing intervals, by adding them together, and reducing them to their lowest terms, or vice versa, is essential to the accurate knowledge of them, the following rule may be added to those already given, in order to remove an ambiguity that might arise; namely, that after the first interval, every other of which a chord may consist, must be reckoned a unit less than their usual denomination; for instance, unless a unit were deducted from each of the two sixths at h, and from the sixth and fifth at i, Ex. 30. the total interval of the former chord would seem to be a seventeenth, and of the latter a sixteenth, instead of the true intervals, a fifteenth and fourteenth.

§67. The Minor Chords Ex. 27. do not admit of the three positions of an added seventh given to the Major Chords at Ex. 28. The first position of the former is the most practical and useful on the Violoncello. The second position or chord of the fifth and sixth is more difficult, on account of its major sixth, together with the minor seventh taken by extension...but the third position, or chord of the third and fourth, is impracticable on this instrument, from the impossibility of introducing a dissonant interval in the position, which reduces itself into a minor sixth and perfect fifth; which, instead of the intended minor chord, is that of the sixth, or first inversion of its relative Major chord as at h, and i, in the following example.

**Ex: 33. TWO POSITIONS of D and A minor, bearing a flat seventh, the third position being impracticable.**

![Diagram of two positions of D and A minor]

First position, Second position,First position, Second position,
§ 68. The terminations or closes, either of a whole piece, or of the smaller portions or phrases, of which it consists, are called Cadences. A cadence consists properly of two chords, in a certain progression or relation, of their fundamental basses, to each other; the preceding chord is called the leading chord, and the close or cadence is said to be made on the latter.

§ 69. Thus the three notes D, C, D, at h, i, k, of the following example may be supposed to be the concluding period of a melody, to which the fundamental bass D, A, D, is assigned. This period is again analysed at l, m, and n, o, into two smaller phrases, with the proper chords; and to the former the fundamental bass D, A, is given, but to the latter A, D. It will be evident that the phrase l, m, with the progression or movement of its fundamental bass, ascending a fifth, is less satisfactory than the phrase n, o, with its fundamental bass, which descends a fifth. The former, therefore, terminating on the fifth of the key, has been called by English and French writers, an imperfect or irregular cadence, by the Italians and Germans, semi Cadenza, or half cadence. And the latter, which concludes on the key, is called a regular or final Cadence.

Ex: 34. PHRASES having imperfect, and perfect cadences in the key of D major.

Fundamental Bass.

§ 70. Again, it may be observed that the leading chord, in the perfect final Cadence, takes the minor seventh, at q, the introduction of which, renders the conclusion on the following chord, more necessary, by making the C to ascend diatonically on D, and the G to descend diatonically on F, as mark'd by the oblique lines; which movements are called the resolution of the dissonant interval of sharp fourth, or tritone, of the former chord, as will be further illustrated in the next chapter.

§ 71. This fifth of the key is also usually called the dominant, from its descent on, and governing the key in a perfect cadence. Its chord has the same properties, in cadences on a Minor key, in which it must equally have a Major third, which occasions the seventh of the minor scale to be raised a semitone, thereby conferring on it a certain desiderative quality, of ascending a semitone on the key, which gives rise to the French appellation of "note sensible," and the English and German term of "leading note" given to the sharp seventh of the scale, and on this account it is, that I have called the first position of a major
perfect chord, or chord of the minor seventh, the Leading position, having the sharp seventh in the highest place; and the third position of the major and minor chord, not bearing the flat seventh, I have called the Final position, as being that with which the key concludes. And thus the Cadences in Minor keys, will be as in the following example.

**Ex: 35.** PHRASES having imperfect and perfect cadences in the key of D minor.

Ex: 35. PHRASES having imperfect and perfect cadences in the key of D minor.

Irregular. | Regular. | Perfect final Cadence.
---|---|---

§72. The following exercise on a phrase in D major, leaving out at first the fundamental note on the fourth string, as at h, in the beginning of the example, will entertain and improve the young performer, by the variety of the bowing it contains, and will give to the hand and fingers, the proper form and facility, to execute in all transposed keys: Let one species of bowing be well acquired, before another is begun, as they proceed gradually from the more simple, to the more complicated. And the minor key Ex: 35 may be occasionally substituted at pleasure. The rotation of bowing (which always begins from right to left, or up bow, mark'd u, and the contrary, or down bow, mark'd d) must be carefully observed, as well as the number of bows employed in each species of arpeggio.

**Ex: 36.** EXERCISE in ARPEGGIO in the key of D major.

No. 1. 4 bows in a bar.

No. 2. 8 bows in a bar.
Ex. 36 Continued.

No. 3.
12 bows in a bar.

No. 4.
6 bows in a bar.

No. 5.
8 bows in a bar.

No. 6.
2 bows in a bar.

No. 7.
Allegro

No. 8.
Allegretto

No. 9.
Allegretto

No. 10.
Allegro
§73. There is another position, or inversion, of the above Cadences Ex: 34 and 35. Frequently used, in which the leading chord, is taken in a second position or chord of the sixth; having the major third in the lowest place, taken by the first finger; and the preceding and following chord may be taken without moving the hand, by placing the second finger on the next semitone and its fifth, and the fourth finger for the major, or third for the minor, on the string above, as represented by directs at the second position Ex: 29. And these positions of both chords, being less conclusive than the former positions, constitute what may be called Medial Cadences.

Ex: 37. Medial Cadences, with or without the dissonant interval.

§74. By changing the fingers the second and fourth, into the first and third, on the same notes in the major chord, as at k and n, a first or leading position is formed, on which new perfect Cadences may be made, in either the major or minor mode. And the discordant interval at m and s, may, in an arpeggio, be added to the chord of the sixth at i and p, as in the following example; in which the interval of the double octave may, in the same manner, be added to the dissonance of the minor seventh.

Ex: 38. Arpeggio of the 5th and the minor seventh.

§75. The converse order of the above chords, form another species of Medial Cadences, as in the following example, where a Cadence in D minor is added, with its medial form, which, notwithstanding of the perfect regular progression of the bass, is imperfect and inconclusive, on account of the leading chord bearing a minor third.

§76. The final and medial cadences above mentioned, and the inverted order of their chords, will be found to prevail in the progression of phrases of harmony; but there are various means of producing variety, and breaking the uniformity of this regular succession; and among others by chords disappointing the expectation, after a leading chord, forming what are called interrupted or suspended cadences, aptly styled by the Italians Cadenze d'inganno.

Ex: 40. INTERRUPTED CADENCES.

§77. A short stroke drawn through a part of the figure, as $\text{♯}$ or $\text{♭}$ as in the above examples, denotes the interval to be a semitone sharper than the clef would imply, viz: a sharp fourth, or sharp sixth; the depression of an interval is marked by a flat or natural before the figure, and sometimes by a small arch over it; as $\text{♯}$ signifies a false or flat fifth.

§78. Although it be a just inference drawn from the nature of intervals, as taken on the Violoncello; that similar cadences or succession of chords must have the same fingering, and are to be executed with equal facility in all keys; yet some perplexity will arise at first, from the circumstance, that every one of the twelve major, and twelve minor keys, may occasionally be expressed by two different names; As in the following Cadences, wherein the C♯ and B♯ of the first, is the Db, and C of the second cadence, and the D♯, and C♯# or CX of the third cadence, is the Eb and D of the fourth.

Ex: 41. CADENCES.

in C♯. in Db. in D♯. in Eb.

or mark the double sharps thus.
§ 76. Instances might be given of Cadences in keys which would appear still more involved and perplexing, merely thro' the apparent intricacy of the signs, or technical terms made use of; tho' the idea or thing in itself be equally simple, both as to clearness of conception, and facility of execution, with any other Cadences, expressed in less complicated signs or terms. It is to be lamented, that this difficulty to Learners, arising from mere perplexity of the language and symbols employed, too frequently discourages them at their first outset, in this as well as in other more important studies; but such terms are however necessary to be known, in order to express our own ideas, or to comprehend those of others; and I shall conclude this subject as I have done that of Intervals, by recommending, what I think the happiest mode that has yet been conceived, of obviating this difficulty in the attainment of musical Science; namely, the musical Games of Miss Young mentioned § 16. The first game consists of two parts, of which the first, conveys the clearest knowledge of all the major and minor keys, as they are delivered in this Essay, and other musical treatises; that is, as far as F♯ of six sharps, and the corresponding key of G♭ of six flats, with their relative minors: but the second part of that Game, proceeding further than other Authors, goes on, in a second circle of keys, from C♯ of seven sharps to B♭ of twelve, and C♭ of seven flats, to D♭ of twelve flats, with their corresponding relative minor keys, and by a few hours practice of this entertaining game, every possible difficulty arising from these perplexing denominations, will be entirely overcome. The third Game is an exercise in the same pleasing manner, on the construction of Cadences in all possible keys major and minor, attended with equal profit and expedition, besides the entertainment as a Game.

CHAP. III. Of the preparation and resolution of discords.

§ 77. It may be proper to premise, that together with the dissonant interval or intervals, contained in a chord, there exists in its full harmony in four parts, its concordant intervals or their inversions; with one or more whereof, the dissonant note itself may be a concord: This will appear from the following analysis of the chord of the minor seventh of A, in which the four different discordant intervals, are more particularly to be attended to.

Ex: 42. Analysis of the concordant and dissonant intervals of the chord of the major third and flat seventh.

See Ex: 14. § 45, 46.

<table>
<thead>
<tr>
<th>Concordant intervals.</th>
<th>Discordant intervals.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inversion.</td>
<td>Inversion.</td>
</tr>
<tr>
<td>3 5 6 4 6 3 6</td>
<td>b7th 20th false fifth</td>
</tr>
</tbody>
</table>
§78. The introduction of the flat seventh which forms the above four dissonant intervals, is rendered more natural and pleasing to the ear, by its previously having constituted a concordant interval in the immediate preceding chord. This is called its preparation; and according to the more rigid rules of harmony is an indispensible condition of its introduction, as a discord; but this may, in modern music, be often dispensed with; and its passing diatonically on the next chord and changing into a concordant interval, is called its resolution, which can never be dispensed with. The two following examples will show its preparation and resolution in two and three parts, where the preparation is pointed out by the binding or slur, and the resolution, by the short oblique lines.

Ex: 43. The flat seventh, second, false fifth, and tritone prepared and resolved in two parts.

Ex: 44. Preparation and resolution of the b7, b3, and tritone in three parts.

§79. The discordant interval is not only frequently introduced without preparation, as in the major and minor final cadences Ex: 94, 95, but the resolution is, on some occasions, when the melody of the parts requires it, allowed to be made in another part, which licence is called in the German School, a changing of the part, as in the following example at i, and l, which may be considered as inversions of the chords and more strict resolution at h, and k; and it may be observed that the licence is taken in order to preserve a diatonic melody in the bass, of the key, its second, and third; and in the upper part, of the third, fourth, and fifth of the key; this being the harmonics assigned to these degrees of the scale in the next chapter; the more strict resolution of the upper part is shown by the direct, and the licence by the oblique lines.
Ex: 45. The 6th unprepared and resolved in a different part.

In D major.

In G major.

CHAP: IV. Of the Harmonies or Accompaniments to the major and minor Scales in all the keys.

§80. Melody has been defined, harmony in succession, and if the existence of the harmonics, and diatonic scale, in the aliquot parts of the string of a musical instrument, mentioned in the preceding theory, will appear to favour this opinion, the following exhibition of the diatonic scale, as the production of three fundamental chords, will give it additional support.

Ex: 46. SCHEME of the derivation of the diatonic Scale from its three fundamentals.

§81. The three fundamental chords are those of the key at i, which produces the third and fifth of the Scale; of the fourth of the key, or Subdominant as it is sometimes called, at h, which produces the fourth and sixth of the Scale; and of the fifth of the key, or dominant at k, which produces the second and seventh of the Scale; which again appear arranged in succession at l, the letters under each degree of the Scale K, D, S, with the directs below, and the figures above them, denote whether they are produced by the Key, Dominant, or Subdominant, and what interval each forms with their fundamentals. The oblique lines drawn from h to i, and i to k, show the progression of the fundamental Basses or generating Sounds, which in their direct and inverse order, are the most usual and natural succession of Chords.

§82. To Accompany the diatonic scale with its appropriate harmonies, in an ascending and descending progression, in all the keys both major and minor, has been ever deemed a proper exercise in learning harmony on key'd instruments; and this Scale has been considered to exist in the Bass, with the following signatures of the chords to be taken.

Ex: 47. The Accompaniment of the Scale on key'd instruments.

Ascending.

Descending.

Fundamental Bass.
§ 83. It would be difficult, if not impossible, to add these harmonies to a diatonic bass on the Violoncello; in lieu of which, the following accompaniment to a diatonic scale, not in the bass, but as a cantilena or melody in the upper part, the bass at the same time keeping the diatonic progression for the most part, is presented as an exercise on the chords and cadences. When it may be observed, that the first chord of the scale being taken as at h, a moving of the hand, or skip of a third, becomes necessary to take the second chord; but if the first chord be taken as at i, the imperfect position of the chord, with the omission of its third, it is attended with this advantage, that the fundamental note is in the lowest part, and the two following chords are in the same position. To the third, fourth, and fifth of the scale at l, m, n, are given the harmonies above mentioned § 79, and Ex: 45, l, the two last chords forming a medial cadence on the key. The chords given to the fifth and sixth of the scale at n, o, are in the same position of the hand (as expressed by the brackets or braces (——) over such positions) forming a medial cadence on the fourth, or subdominant, and those given to the seventh and eighth, are also in one position, forming a final cadence on the key, in the ascending Scale; and an imperfect or half cadence, on the fifth or dominant in descending, which therefore does not admit of the dissonance before given to it.

Ex: 49. Accompaniment of the Scale in G major.

[Diagram of musical notation showing different endings in one position.]
§84. In the accompaniment of the seventh and sixth of the descending scale at \( s, t \), the disagreeable effect of the consecutive fifths at the asterisms, perhaps offensive even to the eyes of a fastidious critic, will not be perceived by the organ which ought to be judge, when it is considered, that these fifths are not heard together, but in single notes in succession, as in the following Arpeggio, which is supposed to accompany the diatonic melody of the scale in the upper line.

Ex: 49. The above chords in arpeggio.

§85. In the accompaniment of the minor Scale, the remarks abovementioned §83 will equally apply in the ascending Scale, there being no difference in the chords except what arises from the minor third of the key, and the minor third of the subdominant, which is the flat sixth of the scale. In the descending scale, the chord of the relative major to the key is given to the seventh, being the best that occurred, tho' the most irregular progression that it contains, with respect to the fundamental bass of the preceding and following chord.
§86. As a further introduction to the foregoing and following scales of all the remaining keys, there are subjoined the chords of the lowest compass of the instrument, in the scales of C, C# and Db, beginning with the third of each Scale in the upper part, and all to be taken on the fourth, third, and second strings, excepting the perfect Cadences added to each scale which employ the four strings.

Ex: 51. Part of the Scale of C.

Ex: 52. Part of the Scale of C#.

Ex: 53. Part of the Scale of Db.

§87. The two scales C# and Db, differ only in the names of the notes; the Cadences at the end of each, are the same with those at Ex: 41, with the difference of the minor seventh in the above, having its resolution pointed out; the open string C, in the second bar of Ex: 52 is of necessity written and call'd B#; as the three first bars are only the inversion (or medial cadence) of the final Cadence at the end. The following table of all the scales accompanied, is in the order of semitones observed in Ex: 26 and 27, and after the first scale, the others are left to be filled up in playing, from the thorough bass signatures, as an exercise to the Learner.
Ex: 52. Accompaniment of the Scale in all the keys.

**Major keys.**

<table>
<thead>
<tr>
<th>No.</th>
<th>First Series</th>
<th>No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>D⁷</td>
<td>D⁷</td>
</tr>
<tr>
<td>Eb</td>
<td>Eb⁷</td>
<td>Eb⁷</td>
</tr>
<tr>
<td>G</td>
<td>G⁷</td>
<td>G⁷</td>
</tr>
<tr>
<td>Ab</td>
<td>Ab⁷</td>
<td>Ab⁷</td>
</tr>
</tbody>
</table>

**Minor keys.**

<table>
<thead>
<tr>
<th>No.</th>
<th>Minor Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>D⁷</td>
</tr>
<tr>
<td>4</td>
<td>Eb⁷</td>
</tr>
<tr>
<td>5</td>
<td>F⁷</td>
</tr>
<tr>
<td>6</td>
<td>G⁷</td>
</tr>
<tr>
<td>7</td>
<td>A⁷</td>
</tr>
<tr>
<td>8</td>
<td>B⁷</td>
</tr>
<tr>
<td>9</td>
<td>C⁷</td>
</tr>
<tr>
<td>10</td>
<td>D⁷</td>
</tr>
<tr>
<td>11</td>
<td>E⁷</td>
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<tr>
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<td>F⁷</td>
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<tr>
<td>13</td>
<td>G⁷</td>
</tr>
<tr>
<td>14</td>
<td>A⁷</td>
</tr>
</tbody>
</table>

Second Series, on the 3rd, 2nd and 1st String.

<table>
<thead>
<tr>
<th>No.</th>
<th>Minor Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>A⁷</td>
</tr>
<tr>
<td>16</td>
<td>B⁷</td>
</tr>
<tr>
<td>17</td>
<td>C⁷</td>
</tr>
<tr>
<td>18</td>
<td>D⁷</td>
</tr>
<tr>
<td>19</td>
<td>E⁷</td>
</tr>
<tr>
<td>20</td>
<td>F⁷</td>
</tr>
<tr>
<td>21</td>
<td>G⁷</td>
</tr>
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<td>A⁷</td>
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<td>23</td>
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<td>24</td>
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<tr>
<td>25</td>
<td>D⁷</td>
</tr>
<tr>
<td>26</td>
<td>E⁷</td>
</tr>
</tbody>
</table>
§88. As an application of the harmony of the scales, to the accompaniment or variation of a given melody, the two following well known Airs are subjoined. In the first, only the six first notes of the scale of A major, No. 15 in the last example, are employed. In the second, in D minor, two Cadences, one in the key, and the other in F, its relative major, constitute its whole compass, viz: C# D, F F; but of these again, the imperfect or semi Cadence D minor, A major, is made at h i, o p, q r, and F C, at m n s u v, the bass and signatures are inserted in the first, and in the next chapter will be given some different chords for the last example.

Ex: 53. The FRENCH AIR. Ah vous dirai je.

Ex: 54. LA FOLIA. a Spanish melody from Corelli's Opera 5th.

CHAP: V. of MODULATION.

§89. Modulation, which in its more general and strict sense, is the transition of sounds taken in succession, is now chiefly understood to express transitions into harmonies connected with each other, but not contained in a given original key. This has for its basis the whole system of practical major and minor keys; the principal relations they maintain to each other, in a linearly descending or ascending progression, as to major keys; and Collaterally, as to their opposite relative minor keys, will be found in the following Scheme.
CIRCLE of all the keys, with their characteristic harmonic intervals.

### Major Keys

<table>
<thead>
<tr>
<th>No.</th>
<th>Key</th>
<th>Harmonic Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>G</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Bb</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Nb</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Eb</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Ab</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Db</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Gb</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>F#</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>B</td>
<td></td>
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<tr>
<td>13</td>
<td>D#</td>
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<td>14</td>
<td>C#</td>
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<td>15</td>
<td>Bb</td>
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<td>16</td>
<td>E</td>
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<tr>
<td>17</td>
<td>G#</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>F#</td>
<td></td>
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</tbody>
</table>

### Relative Minor Keys

<table>
<thead>
<tr>
<th>No.</th>
<th>Key</th>
<th>Harmonic Intervals</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>A</td>
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</tr>
<tr>
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<td>D</td>
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</tr>
<tr>
<td>5</td>
<td>G</td>
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</tr>
<tr>
<td>6</td>
<td>C</td>
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<td>7</td>
<td>F</td>
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<tr>
<td>8</td>
<td>Bb</td>
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</tr>
<tr>
<td>9</td>
<td>Eb</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Ab</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Db</td>
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</tr>
<tr>
<td>12</td>
<td>Gb</td>
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</tr>
<tr>
<td>13</td>
<td>F#</td>
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</tr>
<tr>
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<td>15</td>
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</tr>
<tr>
<td>21</td>
<td>B</td>
<td></td>
</tr>
</tbody>
</table>

Enharmonic keys are indicated by italics.
§90. That the ear is a sufficient guide in directing the thumb or finger, in a descent by perfect fifths, will appear from the two following examples, the first whereof is introductory to the second, by showing more simply the place of the thumb on the second and third strings, which may be compared with the more usual fingering of the same notes in the lower part of the instrument; the directs and small letters in the first example, show on what note the thumb is placed on the first string; namely on the perfect fifth of the respective keys.

Ex. 56. Position of the thumb for the twelve Keys Major and Minor.

Ex. 57.

§91. The young practitioner must not ascribe it to any imperfection in his ear, if he should find the open string D at the end of his modulation too sharp; this being owing to the incommensurability of the Octave by the intervals of perfect fifths (Ex 10 § 29, 30.) The ideal transition from G♭ to F♯ called an enharmonic modulation, is supposed by theorists to make up the deficiency of this small interval; but experiment and practice have shown, that there is no rule or guide for the ear to make any difference in this case; and that the note must be held in the same place. And indeed if the expectation and judgement of the ear are to give the law, as they evidently appear to do, in Cadences and modulations of fundamental progressions, the more deviation is made from the perfection of the fifth, so will the effect be proportionably disagreeable and offensive.
§92. The very same circle of modulation, with the appropriate harmonies, can be made in the lower part of the instrument, by perfect regular Cadences, and taking alternately the discord of the Triton, and false fifth, on the leading chord, and these in the same position of the hand; then moving the hand back to the next tone, the last discord is resolved as in passing from the fourth to the third of the foregoing descending Scales; And this movement of the hand on the perfect chord of Gb or F# is made by passing to that note on the fourth string, on which the lowest sound is afterwards taken to the end of the following example.

Ex: 58. MODULATION by fifths descending.

§93. When the lowest note of the chords of $G$ and $F$ is taken on the third string, the fundamental, which is a fifth below, on the fourth string may be added. As a further illustration of the circle of modulation the following example is added, wherein the resolution of the false fifth is made by the licence of changing the part §79 in the same position together with the false fifth to the following chord, which resolves by moving back the hand one tone on the chord of the $G$. 
§94. The contrary fundamental progression of ascending by fifths, is performed by introducing the characteristic sharp (or natural if from any key with flats) of the new key to be entered into, as a tritone on its leading chord at No. 1 and 2 of the following example; the descending progression in the foregoing examples, having been accomplished by introducing the characteristic flat, or natural of the new key, as a false fifth, on the leading chord. Another more convenient mode for the instrument, is to introduce on the chord of the sixth, a sharp on the fourth of the key, making a false fifth, resolved in descending, as at No. 3 and 4 of the following example.

Ex: 60. MODULATION by fifths ascending.

No. 1.  No. 2.  No. 3.  No. 4.

C.  D.  G.  G.  A.  D.  C.  A.  D.  G.  E.  A.
§95. Passing into a Minor, from its leading Chord, thence into its relative major, and by a medial cadence, into the fourth of such relative major, and inverting that succession, by beginning with the latter and terminating in the first leading chord; is performed with the utmost facility in one position of the hand, as in the following examples; where No. 3 is only a transposition a semitone lower, and No. 4 a semitone higher than No. 1. and by changing the fingers on the last chord, it may become a new leading chord as at No. 5, and produce other modulations, which might also be added to No. 1 and 3 by the same means.

Ex: 61.

§96. The positions and changes of the fingers, in the above modulations, being extremely simple and easy, the learner will derive little advantage from the mere practice, without digesting, reflecting on, and comparing, the keys major and minor, they go into, and their relations to each other in the general system, as leading Chords, Cadences, or otherwise. The Chords in this example, being all taken on the three upper strings, Corresponding Chords, in the same positions, on the three lower strings, will give modulations in keys, all respectively having one sharp less, or one flat more, as their signature.—And the industrious practitioner will much improve himself by writing out the lower note only of each chord, with the thorough bass signature or figures, distinguishing the major and minor chords by the proper sharp, flat, or natural, when such may differ from the signature at the clef.

§97. Similar modulations with No. 1, and the addition at No. 5, may be also made on every other leading position in the system, beginning with that of D, as in Ex: 58. and the relative minor and majors, and progressions thence arising, being observed, or written down, a more clear and extensive view of the whole system, and relations of its parts will thence arise.

§98. Transitions more gradual, and therefore more natural and pleasing, may be made from major to minor, and vice versa, by ascending and descending a few degrees of the scale, nearly in the succession of the Air in D minor Ex: 54., which may be transposed one tone higher, as at Nos. 4, 5, 6 in the following example, or a semitone lower, as at Nos. 7, 8, 9; and the last chord on which a final cadence is made, may be again assumed, and the modulations at No. 10 follow, in corresponding keys, at the end of the cadences No. 3 and 6, as they do at the end of No. 9.
§99. Or the progression No. 1 may be varied, and pass into the minor of F, and afterwards continue in its relative major of Ab, by keeping in the degrees of its Scale, ascending and descending, and pass into the corresponding enharmonic key, as in the following example.

Ex: 63.

§100. It were almost endless, to point out the variety of progressions, which might be introduced on this principle, of degrees of the Scale, Cadences, and the reciprocal relations of major and minor. There are, however, occasionally gradations of more minute intervals introduced on the bass, and the other harmonics of a chord; to which the following variation of the Air Ex: 54. alluded to § 88 may serve as an introduction— the varieties at a, b, c, are substituted for the more simple fundamental progression at n, o, p, and that at d, for the progression at v, of the former Ex: 54.

Ex: 64. Variation of the harmony of Ex: 51.
Of the CHORD of the DIMINISHED SEVENTH.

§ 101. From the Analysis of the chord of the diminished seventh, (mentioned above Ex.15 § 50) in the following example, it appears to consist of two false fifths, as at c and d, and that it cannot be taken in any position or inversion but that at h, which is an interval of a semitone less than the preceding chord of the flat seventh of B flat at g; but on account of its resolving on the chord of C minor, whilst the former must resolve on E♭, it is not a derivative of that chord, but seems a substituted chord for the leading chord of G, as at a, which fundamental does not appear in any of its intervals at b; the fingering at i, is more natural and easy than at h, being founded on the second position of the chord of G, at i, with the difference of the diminished seventh at m; and it is remarkable, that the places of the three intervals, exactly correspond to those of the major sixth and false fifth, at n, which however are not only different in their denomination, but resolve on a very different chord as at r.

Ex: 65. ANALYSIS, and COMPARISON of the DIMINISHED SEVENTH.

§ 102. The following example will show in what manner this chord may be introduced, and disposed of; In No.1 is contained the most variety; but No.2 is the easiest position for the change of fingers. And the next example (Ex: 67) shows its application on all the minor keys, beginning the first series with the open string, which in that chord must be denominated B# as it passes or resolves on C#: the second series begins with the second line, with the third open string, which for a similar reason must be written F#.

Ex: 66.

Ex: 67. CADENCES in all the Minor keys by the diminished Seventh.
Of the CHORD of the sharp fifth.

§103. The perfect chord, is often succeeded, by a dissonant interval of a minor semitone on its fifth, and this in three positions of the chord, without supposing any new fundamental progression, or bass to the chord; with this alteration, it may pass into the chord of the fourth below its fundamental, or into that of its own relative minor, in each of the three positions as will appear in the following example, where the chord appears so regularly to correspond with its preceding perfect chord, that a transposition of it into other keys is unnecessary. In the analysis of this chord, in the first four bars, the intervals will appear when taken on the instrument to be all minor sixths, which in the full chord in the fourth bar would seem to amount to an eighteenth; but when it is considered that the first interval is only a fifth and the unit deducted from the other two sixths (§66) the whole interval is only a fifteenth or double octave.

Ex: 68. POSITIONS of the 5 of D, with its modulations into G major, and B minor.

Of the CHORDS by SUPPOSITION.

§104. These have been already mentioned in §52, 53, 54, and Ex: 15 and 16. The chords of the ninth, and thirteenth, do not appear to me, in any form or inversion in which I can place them, to be adapted to the construction of this instrument, the derivation to which I have referred the diminished seventh Ex: 15 m and Ex: 65 a, being merely hypothetical. The chord of the eleventh, however, may be introduced, with much effect, in concluding phrases like the following; in the first example, it is in the uppermost of three parts only, in the second it is the third interval from the bass, having another part above it; and in both examples it resolves on the tenth of the subsequent chord — its thorough bass signature is 4.

Ex: 69. The chord of the ELEVENTH in all the keys in three parts.
§105. With the additional chords in Ex: 65, and in the subsequent examples, the foregoing, and other modulations, may be occasionally varied and enrich'd, and when the number of keys major and minor in the above Circle Ex: 55, and the different manner in which they may precede and follow each other, is considered; the combinations may be said to be almost endless.

Two treatises of considerable length have been published in this Country exclusively on Modulation, the first by Geminiani, the second many years after by Frike, they are both adapted for key'd instruments, but do not contain any general or settled principles on which the succession of chords is founded.

§106. The transitions as to fingering, being so very easy, the young Performer to derive any advantage from his practice, must be careful to have a previous knowledge of their relations and names, to enable him to pass with propriety, from one to the other; it will much contribute to a compleat understanding of the system, to be able readily to reduce keys with sharps, to correspondent flat signatures, and vice versa, which is done by taking the complement of the given flats or sharps to 12. (as Eb major of three flats, corresponding with D♭ of nine sharps) Also to remember that the leading chord in a minor key, has four sharps more when it becomes a key, or four flats less, than such minor key, and consequently that G♯ minor, five sharps, stands in that relation to Eb, thro' the medium of its leading chord, which being 9 sharps, corresponds with, or is the enharmonic to 3 flats; and again, that B major, is relative to the same G♯ minor; and the relations of other keys may be discovered in the same manner.
§107. A want of exercise occasions these almost merely verbal distinctions, to appear matters of profound and intricate science, which a very little well directed practice, would however easily acquire; and justice to uncommon ingenuity induces me again to recur to the musical Games, twice before introduced, and to recommend them as the safest clue to get at, all times, the almost infinite meandrings of this labyrinth of Modulation. After two interesting Games (play’d by dice, inscribed with the thorough bass figures or signatures) on the accompaniment of the Scale, and the resolution of the more unusual and chromatic discords, the last Game, divided into three parts, is on the important exercise of Modulation, which, tho’ it be now simplified by these Games to the comprehension of ordinary proficient; may be said without any exaggeration, to afford entertainment, even to astonishment, to the greatest masters of harmony.

CHAP: VI. OF SUSPENSIONS and ANTICIPATIONS.

§108. When one or more notes of a chord, in an unaccented part of a bar are prolonged or continued on the accented part of the bar, on a bass, to the harmony of which they do not belong; the latter chord or harmony thereby formed is called a Suspension; and the dissonant interval it contains, is resolved on the next unaccented part of the bar.

§109. At No. 1 of the following example is shown, how the phrase at a, is varied in its harmony at b, by the Suspension on the accented part of the bar, which is resolved on the unaccented part of the same bass, as represented by the thorough bass signature.

[Note: The text continues with a detailed explanation of the musical example, including the resolution of the chord and the dissonant interval, and concludes with a reflection on the merits of the Games, including a quote from a reviewer and the author’s own views on the efficacy and utility of the Games.]
ced in succession. At No. 2, are eight similar phrases, on the different degrees of the scale, each
taken in one position of the hand, where it may be remarked that the minor sixth is express by m,
and the major sixth by M, and the suspending note in a small character. The 6th, 7th, and 8th phrases,
are similar to the 2d, 3d, and 4th, in their intervals, that is, the 2d to the 6th &c. and the positions
or shifts on the instrument are similar; that is, the 2d and 6th phrase are on the half shift, the 3d
and 7th on the whole shift, and the 4th and 8th in the final position of G, being in the relation
of perfect fifths to each other.+

Ex: 71.

No. 1. a b

No. 2. 1st 2d 3d

No. 3.

§110. These eight phrases are written in the usual way at No. 1 in the following exam-
ple, with the Signature to the bass; at No. 2 a, is shown in small notes, in what manner the more
figurative expression of the suspension and resolution at b is derived from the more simple mode
at No. 1, and here it is to be observed that each phrase ends with the concord of the sixth, and the hand
is to shift to each succeeding phrase on the unaccented or last note of every second bar: at No. 3 is
another Variety on the same positions.

Ex: 72.

No. 1.

No. 2.

No. 3.

Andante + These positions or shifts, are illustrated by the easiest examples of popular Airs in my
Theory and practice of fingerimg the Violoncello 2nd edition

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§ 111. Much in the same manner, the more regular sequence of the seventh suspending the sixth, in a descending progression, is performed, only that the hand must shift its position on the same note at the beginning of every bar. The variation in passing notes at No. 2 b, is explained in small characters at a, and here the hand shifts on the last note of each bar.

Ex: 73. No. 1.

§ 112. The preparation of the discordant interval of the second, and its resolution into the third, are evidently Inversions of the suspension and resolution of the seventh into the sixth, tho' not usually called a suspension. This will appear from comparing No. 2 with No. 1 of the following example; and from the first part of Ex: 43, whence I apprehend a decisive argument may be drawn, that the occasional introduction of chords as suspensions, is altogether different and foreign to their construction as chords, which respects the ascertaining their just intervals; as will appear from the introduction of the minor seventh in Ex: 43, whilst their suspensions respect only their use, and manner of introduction, in particular circumstances. Yet on no better grounds than confounding this distinction, has the existence of the chords alluded to § 52 been attempted to be denied, and the chords discarded as phantoms.

§ 113. No. 3 of the following example is an octave higher than No. 2, and taken in a position with the thumb, which is best calculated for the descending series, No. 4; the difficulty of which arises from the thumb being obliged to descend diatonically on the first note of every bar, for the discordant interval of the second, which, in its turn, is resolved by the second finger moving back, on the last, or unaccented part of each bar, after the first. The variety by passing notes at No. 5 b, is explained at a.
§114. When part, or a note or notes of a succeeding chord, is introduced on the bass of an immediate preceding chord, on the unaccented part of the bar, it is called an Anticipation. The difference between which and Suspension, may be illustrated by the following example, at N°1, N°2, N°3 and N°4, and the ascending sequence of fifths and sixths at N°6, which is often used, will appear to be framed on the principle of Anticipations, by comparing it with N°5, by which the faulty progressions and bad effect of the consecutive fifths, are corrected and avoided.

Ex: 75.


Ex. 75.

Engraved by Joseph Caulfield.